HOMEWORK 8 - ANSWERS TO (MOST) PROBLEMS

PEYAM RYAN TABRIZIAN

Section 3.10: Linear approximations and differentials

3.10.2. $L(x) = x - 1$ (because $\ln(1) = 0$, and $\frac{1}{1} = 1$)

3.10.11.

(a) $dy = (2x\sin(2x) + 2x^2\cos(2x))dx$

(b) $dy = \frac{1}{\sqrt{1+t^2}}\left(\frac{t}{\sqrt{1+t^2}}\right)dt = \frac{t}{1+t^2}dt$

3.10.15.

(a) $dy = \frac{1}{10}e^{\frac{x}{10}}dx$

(b) $dy = \frac{1}{10}(0.1) = 0.01$

3.10.21. $\Delta(y) = y(5) - y(4) = \frac{2}{5} - \frac{2}{4} = -\frac{1}{10} = -0.1$
$dy = -\frac{2}{4^2}(1) = -\frac{1}{8} = -0.125$

3.10.25.

$$(8.06)^{\frac{2}{3}} \approx L(8.06) = 8^{\frac{2}{3}} + \frac{2}{3}(8)^{(} - \frac{1}{3})(8.06 - 8) = 4 + \frac{1}{3}(0.06) = 4 + 0.02 = 4.02$$

Here we used the fact that $f(x) = x^{\frac{2}{3}}$ and $a = 8$

3.10.35. $l = 2\pi r = 84$, so $r = \frac{84}{2\pi} = \frac{42}{\pi}$. We know $dl = 0.5$, so $2\pi dr = 0.5$, so $dr = \frac{0.5}{2\pi} = \frac{1}{4\pi}$

(a) $S = 4\pi r^2$, so $dS = 8\pi r dr = 8\pi\frac{42}{\pi}\frac{1}{4\pi} = \frac{84}{\pi}$. Also the relative error is $\frac{dS}{S} = \frac{8\pi r dr}{4\pi r^2} = \frac{2dr}{r} = \frac{1}{2\pi} \times \frac{\pi}{42} = \frac{1}{84} \approx 0.012$

(b) $V = \frac{4}{3}\pi r^3$, so $dV = 4\pi r^2 dr = 4\pi\frac{42^2}{\pi^2} \times \frac{1}{4\pi} = \frac{1764}{\pi^2} \approx 179$. Also the relative error is $\frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = \frac{3dr}{r} = \frac{\frac{3}{4\pi}}{\frac{42}{\pi}} = \frac{3}{168} = \frac{1}{56} \approx 0.018$

3.10.40. $dF = 4kR^3 dR$, so:

$$\frac{dF}{F} = \frac{4kR^3dR}{F} = \frac{4kR^3dR}{F} = \frac{4kR^3dR}{kR^4} = 4\frac{dR}{R}$$

And when $\frac{dR}{R} = 0.05$, $\frac{dF}{F} = 4(0.05) = 0.2$

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Section 3.11: Hyperbolic functions

3.11.9.

$$\cosh(x) + \sinh(x) = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = \frac{e^x + e^{-x} + e^x - e^{-x}}{2} = \frac{2e^x}{2} = e^x$$

3.11.11. Just expand out the right-hand-side and use the fact that $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\cosh(y) = \frac{e^y - e^{-y}}{2}$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(y) = \frac{e^y - e^{-y}}{2}$

3.11.15.

$$2\sinh(x)\cosh(x) = 2\frac{e^x - e^{-x}}{2}\frac{e^x + e^{-x}}{2} = \frac{2}{4}(e^x - e^{-x})(e^x + e^{-x}) = \frac{1}{2}(e^{2x} - e^{-2x}) = \frac{e^{2x} - e^{-2x}}{2} = \sinh(2x)$$

3.11.21. $\sinh(x) = \frac{4}{3}$ (use the fact that $\cosh^2(x) - \sinh^2(x) = 1$ and the fact that $\sinh(x) > 0$ when $x > 0$).

Then you get $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{\frac{4}{3}}{\frac{5}{3}} = \frac{4}{5}$, $sech(x) = \frac{1}{\cosh(x)} = \frac{3}{5}$, etc.

3.11.23(a). 1 (use $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ and factor out e^x from the numerator and the denominator)

3.11.26. This is very similar to example 3 on page 257. However, there is a subtle point involved, check out the document 'Subtle point in 3.11.26' for more info!

3.11.29(a)(b). This is also very similar to example 4 on page 257. For (a), use the fact that $\cosh(\cosh^{-1}(x))^2 - \sinh(\cosh^{-1}(x))^2 = 1$ and $\cosh(\cosh^{-1}(x)) = x$. Also, you'll need to fact that $\sinh(\cosh^{-1}(x)) \geq 0$ (and this is because $\cosh^{-1}(x) \geq 0$ by definition, and $\sinh(x) \geq 0$ if $x \geq 0$). (b) is even easier, use the fact that: $1 - \tanh(\tanh^{-1}(x))^2 = sech(\tanh^{-1}(x))^2$ and $\tanh(\tanh^{-1}(x)) = x$.

3.11.31. $f'(x) = \sinh(x) + x\cosh(x) - \sinh(x) = x\cosh(x)$

3.11.39. $y' = \frac{1}{1+\tanh^2(x)}(sech^2(x)) = \frac{sech^2(x)}{1+\tanh^2(x)}$

Section 4.1: Maximum and Minimum Values

4.1.6.

- Absolute maximum: Does not exist (**NOT** 5)
- Absolute minimum: $f(4) = 1$
- Local minimum: $f(2) = 2$, $f(4) = 1$
- Local maximum: $f(3) = 4$, $f(6) = 3$

4.1.7, 4.1.10. Ask me about that during office hours!

4.1.38. $-1, 1$ (g' does not exist), 0 (makes $g'(c) = 0$)

4.1.39. 0 (F' does not exist), 4, $\frac{8}{7}$ (makes $F'(c) = 0$)

4.1.51. Candidates: $f(-2) = 11$, $f(3) = 66$ (endpoints), $f(0) = 3$, $f(-1) = 2$, $f(1) = 2$. Absolute maximum: $\boxed{f(3) = 66}$, Absolute minimum: $\boxed{f(-1) = f(1) = 2}$

4.1.57. See attached document 'Solution to 4.1.57'